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Parametric Value at Risk

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Abstract

With Value at Risk modelling growing ever more complex we look back at the basics: parametric value at risk. By means of simulation we show that the parametric framework can provide useful risk estimates when used in the right way. We show the danger of relying on historical data and propose two relatively simple adjustments that can benefit the reactivity of the parametric model.

Introduction

In the field of risk management there is an every growing range of techniques available for estimating value at risk (VaR) varying from the very simple to the very complex. The majority of techniques can be categorized as one of three methodologies: analytical/parametric, historical simulation and Monte Carlo simulation. In this short paper we focus on the parametric approach and test by means of simulation to what extent this relatively simple model is able to predict the value at risk of long only equity portfolios.

Methodology

The data used in this simulation consist of daily stock data traded on the NYSE, NASDAQ or AMEX for which there is a complete time series for the year 2000-2010. From this universe of stocks we create ten thousand randomly selected portfolios each containing twenty equally weighted stocks. Model estimation is done using three years (756 days) of historical data and the interpreted results are out of sample.

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Simulations

Simulation 1

The daily 99% VaR is predicted with the following parametric model

$$VaR_{1-\alpha} = z_\alpha * \sigma_p$$
$$\sigma_p = w' \, \Sigma \, w$$

Where:

- w' is a row vector of the asset weights
- Σ is the covariance matrix of the asset returns
- z_α is the left-tail α percentile of a normal distribution

Simulation 2

In our second simulation we apply an exponentially weighted covariance matrix in order to adapt more quickly to changes in market conditions. Weighting will be done with a decay factor of 0.97 which gives the most recent observation a weight of roughly 3% as opposed to 0.13% when weighting each observation equally. In fact, a decay of 0.97 means the returns have a half-life of about 23 days and results in the 75 most recent observations making up for roughly 90% of the weighting.

$$\sigma_{ij}^{exp} = \sum_{t=1}^{T} \frac{(r_{i,t}-\bar{r}_i)(r_{j,t}-\bar{r}_j)\lambda^{t-1}}{\sum_{t=1}^{T} \lambda^{t-1}} \quad \text{and} \quad \bar{r} = \frac{\sum_{t=1}^{T} r_{i,t}\,\lambda^{t-1}}{\sum_{t=1}^{T} \lambda^{t-1}}$$

Where:

- λ is a decay factor between 0 and 1
- r is a simple return adjusted for dividends

Simulation 3

So far our simulations rely on the assumption that the returns of financial time series are normally distributed without any asymmetry (skew) or fat tails (kurtosis). Empirical observations tell us this assumption very rarely holds and can result in a significant underestimation of risk. In an attempt to prevent this we adjust the normal distribution for skew and kurtosis by applying the Cornish-Fischer (CF) expansion on the z-score (z_α). This is done on a daily basis where the degree of skew and kurtosis is estimated from a 1 year rolling window.

$$z_{CF\,\alpha} = z_\alpha + \frac{1}{6}(z_\alpha^2 - 1) * \gamma + \frac{1}{24}(z_\alpha^3 - 3z_\alpha) * \kappa - \frac{1}{36}(2z_\alpha^3 - 5z_\alpha) * \gamma^2$$

Where:

- z_α is the left-tail α percentile of a normal distribution
- γ is the skew in the distribution (third moment)
- κ is the excess kurtosis (fourth moment - 3)

Simulation 4

As a final simulation we adjusted the standard model with the changes made in both simulation 2 and 3.

Please note that no attempt has been made to optimize the observation period, decay factor or portfolio selection.

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Results

Results

As a crude measure of model performance we calculate the percentage of backtest failures (BTFs) for all simulated portfolios over the entire out of sample period. Because we are estimating a 99% VaR we can expect this number to fail 1% of the time. Anything higher indicates risk was underestimated; anything lower indicates that risk was overestimated.

%overshootings	99% VaR	99% CF VaR
noDecay	2.50%	1.78%
Decay	1.48%	1.01%

Table 1. - Overall percentage of back test failures

The standard parametric VaR of simulation 1 results in a disappointing 2.50% of BTFs. Considering this number is an average across both time and portfolios indicates that risk is even more underestimated for individual portfolios and shorter observation windows. Applying more weight to recent observations (simulation 2) considerably improves the VaR estimate as the number of BTFs drop from 2.50% to 1.48%. Adjusting the z-score in simulation 3 with the help of the Corner-Fisher expansion also benefits the original model as the number of BTFs drop from 2.50% to 1.78%. However, both adjustments fail to produce satisfactory results. It is only in our final simulation we see a number of BTFs that is in line with expectation.

It the appendix the back test results for one of the simulated portfolios is shown. Here the adaptive nature of an exponentially weighted covariance matrix together with the corrective effect of the CF expansion can clearly be seen.

To get a better picture of model performance over time we next plot the percentage of BTFs by using a rolling window of one year.

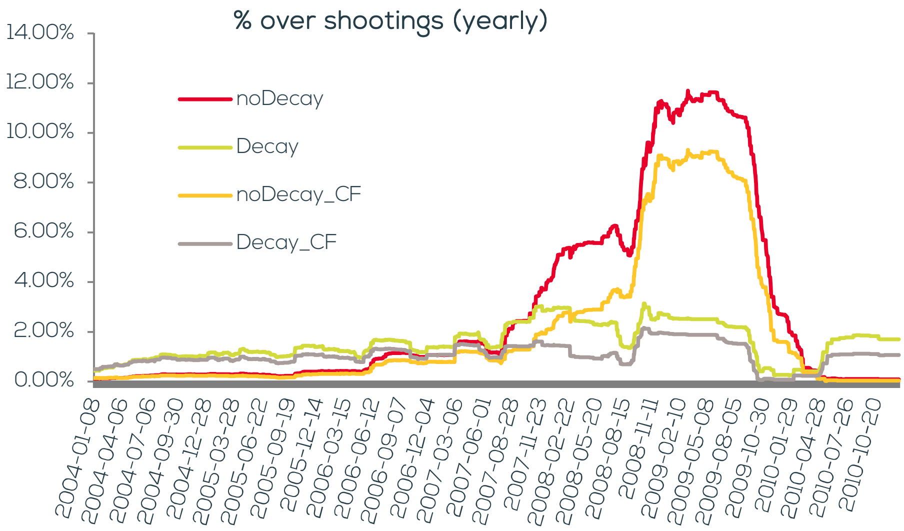

Figure 1 – Percentage of yearly back test failures / over shootings

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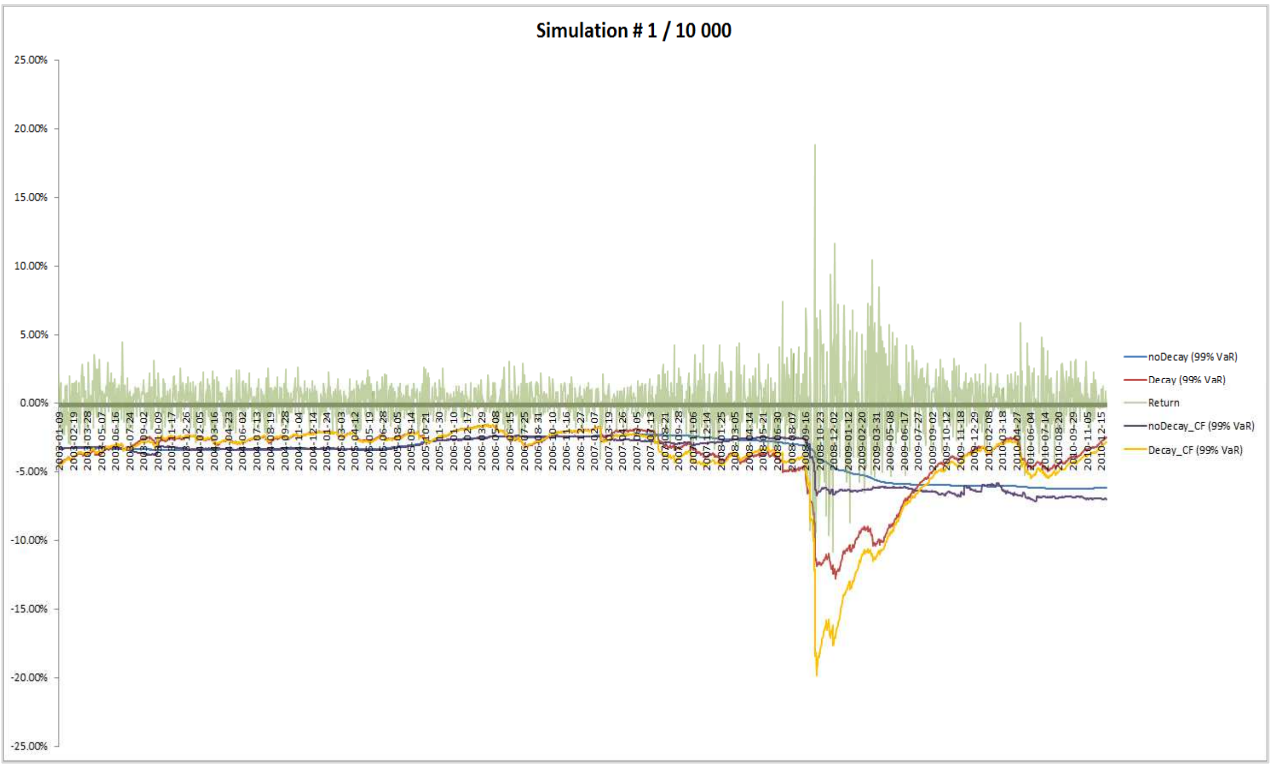
Conclusion

The red and yellow lines clearly shows that the percentage of BTFs is not stable without the use of a decay function as it is very rarely at the expected level of one percent. Together with an unacceptable maximum of 11.70% on the 5th of March 2009 makes this a very poor performance and suggests the model is without much practical use. In contrast, the simulation with a decay function is much more stable and shows a maximum of 3.14% on the 1st of October 2008. Not only is the maximum much lower it also occurs much sooner, indicating that (perhaps not surprisingly) the estimated VaR adapts much quicker in times of financial crises. When applying the CF expansion (green & grey line) we see that in times of growth and low volatility (2004 – 2006) there is not a big difference in terms of BTFs. This is no wonder as such market conditions go hand in hand with relatively low kurtosis and skew, resulting in almost no CF adjustment. It is in times of crises that the CF expansion makes a difference. The percentage of BTFs is slightly more stable and shows a maximum of 2.15% on the 15th of October 2008.

Conclusion

What these simulations show is that it is possible to construct a useful but simple model for estimating value at risk for equity only portfolios. In the case of the parametric model we found it was necessary to give more weight to recent observations by introducing a decay function and to relax the assumption of normally distributed returns with the help of the Cornish-Fischer expansion. Although it is very likely that the choice of other input parameters and observation windows can provide similar or better results, this paper shows that the parametric value at risk framework is not to be underestimated when used in the right way and should remain part of every risk manager's toolbox.

Appendix



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